Journal of Mechanism and Institution Design
ISSN: 2399-844X (Print), 2399-8458 (Online)
DOI:10.22574/jmid.2022.12.005

A REGULATORY ARBITRAGE GAME: OFF-BALANCE-SHEET LEVERAGE AND FINANCIAL FRAGILITY

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ABSTRACT

This study examines a simple banking system in a game-theoretic framework wherein banks act as self-interested agents to maximize leverage at the expense of overall financial stability. The resultant strategic inefficiency raises concerns about how banks manage the "financial stability" good, which is appropriated into a "tragedy of the commons." We conceptualize the inefficiency using the price of anarchy introduced by Koutsoupias & Papadimitriou (2009). We seek the optimal regulatory framework that minimizes the price of anarchy or the degree of financial fragility.

Keywords: Financial fragility, congestion games, price of anarchy, best response potential.

JEL Classification Numbers: G21, G28, D53, C72.

1. INTRODUCTION

SINCE the early 2000s, banks and financial institutions have constructed or employed new instruments to increase leverage without violating regulatory rules. These instruments might not be readily evident on balance sheets

This work has been partly supported by the University of Piraeus Research Center. I am grateful to the editor and two anonymous referees for their helpful comments and constructive suggestions.

yet still expose financial institutions to credit risk. Typically, they include letters of credit, guarantees, operating leases, CDO's, swaps, and OTC derivatives. These off-balance sheet instruments are considered when assessing banks' exposure to credit risk but are subject to different treatment because of their special character.

Such off-balance-sheet leverage could allow banks to transfer credit risk to investors and clear room for new investment opportunities. However, financial institutions, especially banks, employ them not for better risk-sharing but to avoid costly capital buffers and circumvent regulatory requirements. Financial institutions mask credit risk from regulators and increase their risk exposures by undertaking off-balance sheet and other securitizations. This "regulatory arbitrage" compounded the 2008 global financial crisis and its aftermath (Acharya & Richardson, 2009).

The literature on "regulatory arbitrage" is far from new. Pavel & Phillis (1987), and Baer & Pavel (1988) find that lower capital ratios or more demanding regulatory capital are associated with higher levels of off-balance-sheet activities. Jones (2000) discusses the techniques used to undertake regulatory arbitrage and the difficulties faced by regulatory authorities. Breuer (2002) addresses the problem of measurement of off-balance-sheet leverage. It discusses the interaction between risk and off-balance-sheet leverage and calculates a modified capital ratio that incorporates the enhanced leverage implicit in off-balance-sheet securities.

Regulators undertake to assure that financial institutions remain sound, especially banks as the backbone of the financial system. However, their primary tools for ensuring financial stability -cash reserve ratios and capital adequacy ratios- are macro-prudential. That is, they are suited to tackle systemic financial risks.

Systemic financial risk is the risk that an event will trigger a loss of economic value or confidence in a substantial portion of financial system (GTen, 2001). In fact, for a financial system of high concentration, the collapse of a single financial institution suffices to trigger a systemic event. Hence, financial regulatory authorities should be able to assess and manage financial risks to maintain the effectiveness of the financial mechanisms. Moreover, financial risks inhibit banks from diverting high-powered money to higher-return investments, and the latter make banks more resilient to economic downturns at the sacrifice of leverage. Those suggest that institution-level issues warrant attention as well.

We introduce a simple banking system in a game-theoretic framework wherein banks act as self-interested agents, maximizing profits at the expense of overall financial stability. As a result, a "tragedy of the commons" emerges in which banks' strategic behavior produces inefficient outcomes. To measure inefficiency, we use the concept of "price of anarchy" introduced by Koutsoupias & Papadimitriou (2009) and employed by Moulin (2007) and Juarez (2006). In a broader sense, the game we study is a congestion game, a class of games that admit an ordinal potential function (Monderer & Shapley, 1996) that assures an equilibrium outcome. We aim to measure the maximum inefficiency occurring at equilibrium and use that information to measure financial fragility. We elaborate on the bounds of strategic inefficiency, viz. Vetta (2002), and Roughgarden (2006, 2012). Calculating the upper bound of strategic inefficiency reveals how detrimental banks' opportunistic behavior can be.

A further extension of the model incorporates bankers who intend to shake up the financial system to pursue speculative profit. Such behavior is often overlooked in theoretical models; nevertheless, it is customary for speculators to increase market liquidity and short the market on the downside. The Byzantine Generals Problem is an appropriate framework for introducing such destabilizing behavior in the financial system, which originally appeared in distributed systems literature (Lamport et al., 2019). The story behind Byzantine generals is a metaphor for a connected network of agents that must reliably communicate a common plan of action. However, among the loyal agents, some traitors undermine the agreement. The question that emerges is how tolerant the network (i.e., the financial system) is of the perverse incentives of "traitors" (i.e., speculators).

The rest of the paper proceeds as follows. Section 2 presents our model and its measure of strategic inefficiency. In addition, it associates the boundedness properties of inefficiency with the literature on generic cost-minimization games. Section 3 extends the model to bankers that benefit by destabilizing the financial system. Section 4 concludes.

2. MODEL

Suppose a one-shot game involving $I = \{1,\ldots,n,n+1\}$ players. The first $n \geq 2$ players are banks and the $n+1$ is a pseudo-player that stands for the financial regulatory authority (FRA). Each bank has a simple balance sheet on which liabilities are deposits (D) and bank's capital. The deposit rate is zero ($r^D = 0$) for convenience. Assets are cash balances and a single asset (A) that generates a positive return ($r^m > 0$). Regard r^m as return on assets, which without loss of generality, we assume is identical across banks. The FRA has a decisive role in the game. As a tool of regulatory policy, it adopts a *capital adequacy ratio* ψ (a percentage of assets to be held in cash) and a *reserve requirement ratio* θ (a percentage of deposits to be held in cash). Banks eventually incur a "regulatory tax" amounting to the opportunity cost of holding reserves and capital that could be invested for a positive return in asset A. Foregone profits for bank $i \in I \setminus \{n+1\}$ attributable to regulatory tax are estimated as

$$RT_i = \psi \cdot A_i \cdot r^m + \theta \cdot D_i \cdot r^m = r^m(\psi \cdot A_i + \theta \cdot D_i).$$

Absent regulation, the bank could invest both reserved deposits $(\theta \cdot D_i)$ and reserved capital $(\psi \cdot A_i)$ in asset A and enjoy with certainty a positive return r^m.

Cost function

We assume that incidents of financial distress occur horizontally during which all banks suffer a haircut of ω percent. Hence, the objective of bank i is to minimize total cost that includes the regulatory tax and ω. We assume two specifications of total cost. First, for bank i and a proper subset of banks $S \subseteq I \setminus \{n+1\}$ the cost function is given by

$$C_i = \alpha_i[r^m(\psi \cdot A_i + \theta \cdot D_i)] + (1-\alpha_i)\frac{\sum_{j=1}^{\#S}(1-\alpha_j)}{n}\omega \cdot A_i,$$

which for $\alpha_i \in (0,1)$ and substituting RT_i becomes

$$C_i = \alpha_i RT_i + (1-\alpha_i)^2\frac{\omega A_i}{n} + (1-\alpha_i)\frac{\sum_{j=1}^{\#S\setminus\{i\}}(1-\alpha_j)}{n}\omega \cdot A_i. \quad (1)$$

Bank i decides to circumvent part $(1-\alpha_i)$ of the regulatory tax by committing off-balance-sheet activities and to remit the remainder α_i. Hence, α_i is the strategic variable of bank i and determines how much regulatory tax it circumvents. The first term on the right in Eq.(1) is the cost of regulatory tax. The second and third terms denote the expected loss from financial distress.

If $S = \{j \in I \setminus \{n+1\} | s.t.\ \alpha_j < 1\}$ is the subset of banks that circumvent some or all of their regulatory tax, the probability of financial distress will be $\frac{\sum_{j=1}^{\#S}(1-\alpha_j)}{n}$ and the haircut for individual bank i will be ωA_i. As the number of evading banks (i.e. $\#S \to n$) and the regulatory tax evasion ($\alpha_i \to 0$) increase the probability of financial distress tends toward 1.

A drawback of this specification is that the bank becomes immune to systemic risk when fully complying with regulation ($\alpha = 1$). We can relax this strong assumption by assuming that all banks bear the cost in case of financial distress. A different specification to accommodate these conditions is

$$\begin{aligned} C_i &= \alpha_i[r^m(\psi \cdot A_i + \theta \cdot D_i)] + \frac{\sum_{j=1}^{\#S}(1-\alpha_j)}{n}\omega \cdot A_i \\ &= \alpha_i[r^m(\psi \cdot A_i + \theta \cdot D_i)] + \frac{(1-\alpha_i)}{n}\omega \cdot A_i + \frac{\sum_{j \neq i}(1-\alpha_j)}{n}\omega \cdot A_i. \end{aligned} \tag{2}$$

Banks reduce the probability of financial distress if they fully comply with financial regulation, but they can be *contaminated* by a financial crunch and suffer its consequences. We call this cost function the *cost with contagion effect*.

Price of Financial Anarchy

From the FRA's perspective the *social optimum cost* is that all banks opt for $\alpha_i = 1$. Doing so makes the overall cost equal to $\bar{\mathbf{C}} = \sum_i C_i((\psi, \theta, \alpha = \mathbf{1})$. *Social optimum cost* is the overall regulatory tax,

$$SOC = \bar{\mathbf{C}} = \sum_i RT_i = RT.$$

At Nash equilibrium overall (social) cost is denoted $\mathbf{C}^* = \sum_i C_i((\psi, \theta), \alpha^*)$. Departures from social optimum can be computed using a coordination ratio, known in game-theoretic literature as the *price of anarchy* (Koutsoupias & Papadimitriou, 2009). For our model, we call it the *price of financial anarchy* (PFA). The ratio of the cost at Nash equilibrium over the social optimum cost ($C^*/\bar{C}$) can be a metric of banking system disobedience to FRA policies.

Definition 1. *PFA is defined as the maximum deviation from social optimum cost for the worst-case equilibrium in the set of equilibria. It is the ratio*

$$PFA = \max_{\alpha^* \in NE} \frac{\mathbf{C}^*}{\bar{\mathbf{C}}}. \tag{3}$$

We anticipate the PFA metric to take values above 1. The following lemma proves the result.

Lemma 1. *PFA* > 1, *when for all banks i*

$$\frac{\omega A_i}{RT_i} \geq \frac{n}{\sum_{\forall j}(1-\alpha_j)}.$$

Proof. Appendix A.

Similar to PFA is the price of financial stability, which measures the deviation from the best-case equilibrium. The two measures coincide in the case of a unique equilibrium.

The Regulatory Arbitrage Game

To exert its stabilizing role in the financial system, the FRA aims to minimize the objective $C_{n+1} = |PFA - 1|$ by choosing appropriate policy parameters (ψ, θ). The game's strategy profile of banks is $\alpha \in [0,1]^n$ given the policy mix of the FRA .

Definition 2. *The regulatory arbitrage game is defined by the tuple*

$$\Gamma = \{I, \{[0,1]\}_{i \in I}, \{C_i\}_{i \in I}, (\psi, \theta)\}.$$

Nash equilibrium

Nash equilibrium emerges when all banks simultaneously minimize their total costs.

Definition 3. *Given policy parameters* (ψ, θ), *a Nash equilibrium is a strategy profile* α^* *such that for all banks i*

$$C_i((\psi,\theta),\alpha^*)) \leq C_i((\psi,\theta),(\alpha_i,\alpha^*_{-i})) \quad \textit{for all} \;\; \alpha_i.$$

Remark. Nash equilibrium always exists. Both strategy sets and cost functions are convex, so a minimum always exists.

The regulatory arbitrage game admits a *best-response potential*. Best-response potential games guarantee Nash equilibrium when each player's cost function is non-linear. The game $\Gamma = \{I, [0,1]^n, \{C_i\}_{i\in I}, (\psi, \theta)\}$ admits *best-response potential* $P : [0,1]^n \mapsto R$ such that

$$\arg\min_{\alpha_i} C_i(\alpha) = \arg\min_{\alpha_i} P(\alpha)$$

Next, we provide the best response potential for our game.

Lemma 2. *The best-response potential function of the regulatory arbitrage game is*

$$P(\alpha) = \sum_i (1-a_i)^2 \frac{\omega A_i}{n}.$$

Proof. Appendix B.

Now we provide the existence of equilibrium.

Proposition 1. *The Nash equilibrium of the regulatory arbitrage game always exists.*

Proof. By Lemma 2 and Proposition 2.2 in Voorneveld (2000) the game has a Nash equilibrium. □

Bounds to inefficiency

The finiteness and the noncooperative character of the game make the equilibrium outcome Pareto inefficient. It is true that for a noncooperative game with a finite and self-interested set of players, there is always a non-equilibrium outcome that is superior in a Pareto sense (Dubey, 1986). Exemplified games are the prisoner's dilemma and the standard Cournot duopoly; nevertheless, the degree of inefficiency remains to be found. For this task, we provide further definitions. We denote $\alpha_{-i} \geq \alpha'_{-i}$ whenever component-wise $\alpha_j \geq \alpha'_j$ for all $j \neq i$. With slight abuse of notation, also let $\alpha_{-i} = \alpha_j$.

Definition 4. *We say that the cost function exhibits decreasing differences if for $a_i \geq a'_i$ and $a_j \geq a'_j$ it is*

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha'_j) \leq C_i(\alpha'_i, \alpha_j) - C_i(\alpha'_i, \alpha'_j), \quad \forall i \in I \setminus \{n+1\}. \tag{4}$$

A game with cost functions that exhibit decreasing differences is *submodular*. Next, we show that bankers' cost functions exhibit decreasing differences.

Lemma 3. *The cost function of banks in the regulatory arbitrage game exhibits decreasing differences i.e., for $\alpha_i \geq \alpha_i'$ and $\alpha_j \geq \alpha_j'$ it is*

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha_j') \leq C_i(\alpha_i', \alpha_j) - C_i(\alpha_i', \alpha_j'), \quad \forall \in I \setminus \{n+1\}.$$

Proof. Appendix C.

Lemma 3 assures that banks always have escalating incentives to circumvent regulatory tax, for the greater the circumvention, the greater is the cost-saving. Next, we prove that the decreasing differences are linear for the second specification of the cost function. Finally, the next lemma says that submodularity is maintained, albeit linearly, under contagion effects.

Lemma 4. *The cost function with contagion effect exhibits linear decreasing differences. That is,*

$$C_i(\alpha_i, \alpha_j) - C_i(\alpha_i, \alpha_j') = C_i(\alpha_i', \alpha_j) - C_i(\alpha_i', \alpha_j'), \quad \forall i.$$

Proof. Appendix D.

Also, important in this analysis is individual bank's ability to affect social cost. Suppose the extreme wherein all banks except bank i comply fully with regulatory policy. That is (ψ, θ), ie $C_i((\psi, \theta), \alpha_i < 1, \alpha_j = 1)$. Such behavior saves costs for bank i with respect to regulatory tax. We define the cost of bank i at unilateral deviation as the *positive pivotal cost* of bank i, denoted PC_i^+. We define *negative pivotal cost* as $PC_i^-((\psi, \theta), \alpha_i = 1, \alpha_j < 1)$, i.e., when bank i unilaterally complies fully with regulatory policy (ψ, θ). In the latter case, bank i bears the full regulatory tax and cost of systemic risk for the case of a cost function with contagion effects. It is straightforward, then, to ascertain that

$$PC_i^+ \leq C_i((\psi, \theta), \alpha = \mathbf{1}) \leq PC_i^-. \tag{5}$$

The following is a permissive assumption.

Assumption 1. $PC_i^- - C_i((\psi, \theta), \alpha = \mathbf{1}) \geq C_i((\psi, \theta), \alpha = \mathbf{1}) - PC_i^+$.

The latter suggests that the additional cost of conforming to regulation when all others act strategically exceeds the cost saved by unilaterally circumventing it when all others conform. The assumption is satisfied for both cost function specifications in the model.

The average pivotal effect of bank i can be attributed by the *average pivotal cost* $APC_i = (PC_i^+ + PC_i^-)/2$ and gives information about the net effect of bankers' unilateral deviations. In the case of cost function with immunization (Eq. (1)) we anticipate that $C_i((\psi, \theta), \alpha = \mathbf{1}) = PC_i^-$, because the bank becomes fully immune to systemic risk and bears only the regulatory tax. Hence, it is $APC_i \leq C_i((\psi, \theta), \alpha = \mathbf{1})$.

Accordingly, we define *total average pivotal cost* as $TAPC = \sum_i APC_i$. Per Topkis (1998) (lemma 2.6.1. p49), we know that the sum of submodular functions is submodular and all properties of the (average) cost function are inherited by total (average) cost.

Proposition 2. *The PFA is bounded from above by* $\max_{\alpha^*}\left\{\frac{2TAPC-SOC}{SOC}\right\}$.

Proof. Appendix E.

Corollary 1. *Under Assumption 2, the PFA is bounded away from 1 and Nash equilibrium is always inefficient.*

Proof. Appendix F.

Proposition 2 indicates that all Nash equilibria are inefficient and that the upper bound indicates how detrimental it can be for banks to undertake off-balance-sheet activities. The higher the upper bound of PFA, the more susceptible the system is to banks' opportunism; nevertheless, we cannot make sure that the upper bound is actually attained. Under Eq. (2), this upper bound has important effects on the negative pivotal cost of banks (PC_i^-). The latter stands for the bank's cost to conform with regulatory policy when all competitors act strategically. The higher the differential in the equation of Assumption 2, the more costly conformance becomes. If we claim strict inequality in Assumption 2, the upper bound is always distant from 1.

Section 3 extends the model to include banks that willfully seek to destabilize the banking system.

3. THE GAME WITH "BYZANTINE" BANKERS

Economies can include willfully destabilizing agents - e.g., short-sellers who manipulate a collapse in stock prices or bond traders who pressure an issue to trigger credit default swaps that they hold. Such trades are not always traceable. Dark pools that allow institutional investors to mask their activity from other market participants account for 14% of US stock trading volume (Buti et al., 2017). Dark pool trading platforms bypass and fragment open markets and disrupt market information.

Once trade masking appears, some bankers may seek to destabilize the financial system. We call these *malicious Byzantine bankers* after the Byzantine generals' problem in the computer network literature. We divide the game with *Byzantine Bankers* into two classes: a proper subset of profit maximizing bankers I^p, and malicious bankers I^m, with $I = I^p \cup I^m$. We assume that each banker in I^m aims to destabilize the system, seeking to maximize the difference $|PFA - 1|$. We anticipate that no Byzantine banker eventually opts for a positive α_i, whatever the cost to their balance sheets, as they profit with off-balance-sheet activities.

Definition 5. *The Byzantine regulatory arbitrage game is defined by the cost minimization game*

$$\Gamma = \{I, \{[0,1]\}_{i \in I}, \{C_i\}_{i \in I^p}, \{C_i\}_{i \in I^m}, (\psi, \theta)\}\}.$$

In this specification, overall social cost includes only costs incurred by profit maximizing bankers, $i \in I^p$. It is legitimate to exclude malicious Byzantine bankers from social cost because we assume they undermine social welfare. Malicious players have a destabilizing role in the economy; they favor an increasing social cost and operate as adversaries to profit-maximizing bankers. Therefore, excluding them from the overall social cost from a regulator's viewpoint is correct. Thus, $\bar{\mathbf{C}} = \sum_{i \in I^p} C_i((\psi, \theta, \alpha = \mathbf{1}|I^m)$. Accordingly, we define equilibrium in the game with Byzantine players, α^*, which we call *Byzantine Nash equilibrium*, and the overall cost to profit-maximizing bankers at equilibrium, $\mathbf{C}^* = \sum_{i \in I^p} C_i((\psi, \theta), \alpha^*|I^m)$.

We keep the information conditions of the game as abstract as possible. In the Byzantine agreement framework, it is customary to assume that we only know the presence of malicious players. Nevertheless, all the relevant information is available to define the price of financial anarchy in this context. Therefore, we modify the PFA to accommodate their presence. In this scenario, the worst-case equilibrium is given by a ratio that is far distant from 1. The following lemma provides a necessary condition for the latter.

Lemma 5. *The (profit maximizing) bank's cost at equilibrium will exceed the social optimum cost whenever*

$$\frac{\omega A_i}{RT_i} \geq \frac{n}{m + \sum_{j \in I^p}(1 - \alpha_j)}.$$

Proof. Appendix G.

By lemma 1, the haircut (ωA_i) exceeds the regulatory tax (RT_i).That condition is likely satisfied when the number of banks (n) is relatively small, assuming so throughout the proof.

Definition 6 (Price of Byzantine Financial Anarchy)**.** *The Price of Byzantine financial anarchy (PBFA) is defined as the deviation from social optimum cost for worst- case equilibrium (in the equilibrium set). The ratio is*

$$PBFA(I^p; I^m) = \max_{\alpha^*} \frac{\mathbf{C}^*(I^p; I^m)}{\bar{\mathbf{C}}(I^p)}. \tag{6}$$

The PBFA captures the suboptimality of the worst-case Nash equilibrium in the extended version of the game with Byzantine bankers, and it is not much different from the PFA in practice. As claimed, we do not calculate the cost of Byzantine bankers in overall social cost. However, we consider their strategic influence on the cost of the remaining (profit-maximizing) bankers. Following Moscibroda et al. (2006) we define the *price of malice* (PoM) that conceptualizes the relative inefficiency for the original game.

Definition 7. *The Price of Malice (PoM) measures inefficiency in the system caused by Byzantine bankers and is given by the ratio*

$$PoM(I^m) = \frac{PBFA(I^p; I^m)}{PFA(I^p)}. \tag{7}$$

The PoM describes the degree of suboptimality resulting from Byzantine bankers. The lower the PoM is, the more tolerant the system is to the presence of malicious participants. Put differently, it measures how much damage is caused by the presence of Byzantine bankers.

Let us now see how detrimental the activity of Byzantine bankers can be to the financial system.

Proposition 3. *PBFA is bounded from above by the ratio*

$$\max_{\alpha^*}\left\{\frac{2TAPC-SOC}{SOC}+m\Gamma\right\},$$

with $\Gamma=\sum_{I^p}(1-\alpha_j)\cdot\frac{\omega A_i}{n}>0$.

Proof. Appendix H.

Interestingly, PBFA can be expressed as the decomposition of the original PFA attributed to the strategic inefficiency of profit-maximizing bankers and the component of financial risks originating with Byzantine bankers. This decomposition might help to assess differing regulatory constructs. For example, we can calculate how discouraging each proposed policy could be for profit-maximizing bankers and how immune the financial system becomes from the actions of Byzantine bankers. When policies primarily target the destabilizing role of Byzantine bankers, it might be more appropriate to use PoM as the indicator.

Corollary 2. *PoM in the Byzantine regulatory arbitrage game is*

$$PoM(I^m)=\frac{m\Gamma\cdot SOC}{2TAPC-SOC}.$$

Proof. The corollary follows directly from the definition. □

The PoM increases when the number of malicious bankers increases or the overall cost of fully complying with financial regulation rises.

4. DISCUSSION

This study addresses the problem caused by profit-maximizing bankers when they try to circumvent regulations and seek extra profit at the peril of system stability. The regulatory arbitrage game is an abstract but powerful framework for addressing banks' strategic considerations. Banks have the opportunity to increase their leverage and amplify profits. So long as these practices impose no cost on bankers and many financial instruments remain unregulated, malevolent motives remain. Drawing upon the "price of anarchy," we introduce the necessary theoretical underpinnings to capture social inefficiency caused by profit-maximizing bankers. To our knowledge, there is no other study that makes use of congestion games to contemplate the incentives of a bank's management in a regulated environment.

The price of financial anarchy is novel and can be used in three respects. First, to assess whether market regulations correct a vulnerability in financial systems; second, to calculate the critical PFA values that make the financial system fragile; third, to pursue a regulation that suppresses it below these thresholds. In a broad sense, financial fragility conceptualizes triggering a financial crisis by an exogenous (small) financial or economic shock. We emphasize that the more unregulated the financial system, the higher is the risk of triggering a crisis. Put differently, Byzantine bankers always seek to circumvent regulations to profit from financial turmoil, and that opportunity emerges in upturns and downturns.

Appendices

A. PROOF OF LEMMA 1

For the arbitrary strategy profile of banks α the cost function takes the form,

$$C_i = \alpha_i RT_i + (1-\alpha_i)^2 \frac{\omega A_i}{n} + (1-\alpha_i)\frac{\sum_{j\neq i}(1-\alpha_j)}{n}\omega \cdot A_i$$

We require for all $i \in I \setminus \{n+1\}$ to be $C_i > \bar{C}_i = RT_i$.

$$\begin{aligned}
\alpha_i RT_i + (1-\alpha_i)^2 \frac{\omega A_i}{n} + (1-\alpha_i)\frac{\sum_{j\neq i}(1-\alpha_j)}{n}\omega \cdot A_i &> RT_i \\
(1-\alpha_i)\frac{\omega A_i}{n}[(1-\alpha_i) + \sum_{j\neq i}(1-\alpha_j)] &> (1-\alpha_i)RT_i \\
\frac{\omega A_i}{n}\sum_{\forall j}(1-\alpha_j) &> RT_i \\
\frac{\omega A_i}{RT_i} &> \frac{n}{\sum_{\forall j \in I\setminus\{n+1\}}(1-\alpha_j)} \\
&> 1.
\end{aligned}$$

□

B. PROOF OF LEMMA 2

For an arbitrary i 's cost function in Eq. 1, the first derivative gives

$$\frac{\partial C_i}{\partial \alpha_i} = RT_i - 2(1-\alpha_i)\frac{\omega A_i}{n} - \sum_{j=1}^{\#S\setminus\{i\}}(1-\alpha_j)\frac{\omega \cdot A_i}{n}.$$

Now define the function $\hat{C}_i = (1-\alpha_i)^2\frac{\omega A_i}{n}$ for which the first derivative is $\frac{\partial C_i}{\partial \alpha_i} = -2(1-\alpha_i)\frac{\omega A_i}{n}$.

Since the domain of α_i is a convex subset of reals and both $C_i, \hat{C}_i$ are quadratic, first-order conditions are sufficient for a minimum. Hence both achieve a minimum for some α_i. It is easily seen that both functions achieve minima for the same α_i. That is,

$$\arg\min_{\alpha_i} C_i(\alpha_i;\alpha_{-i}) = \arg\min_{\alpha_i} \hat{C}_i(\alpha_i;\alpha_{-i}).$$

Now we define the function,

$$P(\alpha) = \sum_{i}^{I\setminus n+1}\hat{C}_i = \sum_{i}^{I\setminus n+1}(1-a_i)^2\frac{\omega \cdot A_i}{n}.$$

The function P is twice differentiable with respect to α_i and strictly convex. Differentiating,

$$\frac{\partial P}{\partial \alpha_i} = -2(1-\alpha_i)\frac{\omega A_i}{n} = \frac{\partial C_i}{\partial \alpha_i}. \tag{8}$$

it follows that

$$\arg\min_{\alpha_i} C_i(\alpha_i;\alpha_{-i}) = \arg\min_{\alpha_i} \hat{C}_i(\alpha_i;\alpha_{-i}) = \arg\min_{\alpha_i} P(\alpha_i;\alpha_{-i}),$$

which makes function $P(\alpha)$ an admissible best-response potential, i.e.,

$$\arg\min_{\alpha_i} C_i(\alpha) = \arg\min_{\alpha_i} P(\alpha).$$

□

C. PROOF OF LEMMA 3

By substituting (1) into the definition of decreasing differences (4), we have for the left side

$$C_i(\alpha_i,\alpha_j)-C_i(\alpha_i,\alpha_j')=\frac{(1-\alpha_i)\omega A_i}{n}\left(\sum_{j\in S\setminus\{i\}}(1-\alpha_j)-\sum_{j\in S\setminus\{i\}}(1-\alpha_j')\right).$$

Similarly, for the right side we have

$$C_i(\alpha_i',\alpha_j)-C_i(\alpha_i',\alpha_j')=\frac{(1-\alpha_i')\omega A_i}{n}\left(\sum_{j\in S\setminus\{i\}}(1-\alpha_j)-\sum_{j\in S\setminus\{i\}}(1-\alpha_j')\right).$$

For $\alpha_i \geq \alpha_i'$ it is always

$$\frac{(1-\alpha_i)\omega A_i}{n}\leq\frac{(1-\alpha_i')\omega A_i}{n}.$$

Hence $C_i(\alpha_i,\alpha_j)-C_i(\alpha_i,\alpha_j')\leq C_i(\alpha_i',\alpha_j)-C_i(\alpha_i',\alpha_j')$. □

D. PROOF OF LEMMA 4

We prove the lemma *mutatis mutandis* following the proof of Lemma 3.

For the left side,

$$\begin{aligned}C_i(\alpha_i,\alpha_j)-C_i(\alpha_i,\alpha_j') &= \alpha_i RT_i+\frac{(1-\alpha_i)}{n}\omega\cdot A_i+\frac{\sum_{j\neq i}(1-\alpha_j)}{n}\omega\cdot A_i\\ &\quad-\alpha_i RT_i-\frac{(1-\alpha_i)}{n}\omega\cdot A_i-\frac{\sum_{j\neq i}(1-\alpha_j')}{n}\omega\cdot A_i\\ &=\frac{\omega A_i}{n}\left(\sum_{j\in S\setminus\{i\}}(1-\alpha_j)-\sum_{j\in S\setminus\{i\}}(1-\alpha_j')\right).\end{aligned}$$

Similarly, for the right side we have

$$\begin{aligned}C_i(\alpha_i',\alpha_j)-C_i(\alpha_i',\alpha_j') &= \alpha_i' RT_i+\frac{(1-\alpha_i')}{n}\omega\cdot A_i+\frac{\sum_{j\neq i}(1-\alpha_j)}{n}\omega\cdot A_i\\ &\quad-\alpha_i' RT_i-\frac{(1-\alpha_i')}{n}\omega\cdot A_i-\frac{\sum_{j\neq i}(1-\alpha_j')}{n}\omega\cdot A_i\\ &=\frac{\omega A_i}{n}\left(\sum_{j\in S\setminus\{i\}}(1-\alpha_j)-\sum_{j\in S\setminus\{i\}}(1-\alpha_j')\right).\end{aligned}$$

Evidently, $C_i(\alpha_i,\alpha_j)-C_i(\alpha_i,\alpha_j')=C_i(\alpha_i',\alpha_j)-C_i(\alpha_i',\alpha_j')$. □

E. PROOF OF PROPOSITION 2

Per Topkis (1998)(Lemma 2.6.1, p. 49), we know a sum of submodular functions is submodular. Therefore, collective cost function $\mathbf{C}=\sum_i C_i$ is submodular. Consider the following profiles: $\bar{\alpha}=(1,\mathbf{1})$ and $\alpha^*=(\alpha_i^*,\alpha_{\mathbf{j}}^*)$ with $\bar{\alpha}$ being the profile of all bankers complying fully with regulation and corresponding to the optimal solution with regard to FRA, and α^* a Nash equilibrium.

By the property of decreasing differences,

$$\begin{aligned}
C_i(\bar{\alpha}_i,\bar{\alpha}_j)-C_i(\bar{\alpha}_i,\alpha_j^*) &\le C_i(\alpha_i^*,\bar{\alpha}_j)-C_i(\alpha_i^*,\alpha_j^*)\\
C_i(\bar{\alpha}_i,\bar{\alpha}_j)+C_i(\alpha_i^*,\alpha_j^*) &\le C_i(\alpha_i^*,\bar{\alpha}_j)+C_i(\bar{\alpha}_i,\alpha_j^*)\\
C_i(\bar{\alpha}_i,\bar{\alpha}_j)+C_i(\alpha_i^*,\alpha_j^*) &\le PC_i^+ + PC_i^- = 2APC_i\\
1+\frac{C_i(\alpha_i^*,\alpha_j^*)}{C_i(\bar{\alpha}_i,\bar{\alpha}_j)} &\le \frac{2APC_i}{C_i(\bar{\alpha}_i,\bar{\alpha}_j)}\\
\frac{C_i(\alpha_i^*,\alpha_j^*)}{C_i(\bar{\alpha}_i,\bar{\alpha}_j)} &\le \frac{2APC_i-C_i(\bar{\alpha}_i,\bar{\alpha}_j)}{C_i(\bar{\alpha}_i,\bar{\alpha}_j)}.
\end{aligned}$$

Applying summation by parts for all banks,

$$\begin{aligned}
max_{\alpha^*}\frac{\mathbf{C}(\alpha_i^*,\alpha_j^*)}{\mathbf{C}(\bar{\alpha}_i,\bar{\alpha}_j)} &\le max_{\alpha^*}\frac{\sum_n 2APC_i - SOC}{SOC}\\
PFA &\le \max_{\alpha^*}\left\{\frac{2TAPC-SOC}{SOC}\right\}.
\end{aligned}$$

□

F. PROOF OF COROLLARY 1

Viz. the proof of Proposition 2 we ask

$$\frac{2APC_i-C_i(\bar{\alpha}_i,\bar{\alpha}_j)}{C_i(\bar{\alpha}_i,\bar{\alpha}_j)}\ge 1,$$

or

$$\begin{aligned} 2APC_i - C_i(\bar{\alpha}_i, \bar{\alpha}_j) &\geq C_i(\bar{\alpha}_i, \bar{\alpha}_j) \\ PC_i^+ + PC_i^- - C_i(\bar{\alpha}_i, \bar{\alpha}_j) &\geq C_i(\bar{\alpha}_i, \bar{\alpha}_j) \\ PC_i^- - C_i(\bar{\alpha}_i, \bar{\alpha}_j) &\geq C_i(\bar{\alpha}_i, \bar{\alpha}_j) - PC_i^+. \end{aligned}$$

Under Assumption 2, Corollary 1 is always true. □

G. PROOF OF LEMMA 5

Denote the cost function of bank i in the game with Byzantine bankers by $C_i^B(\alpha_i, \alpha_j; \alpha_{\mathbf{m}} = \mathbf{0})$, where vector $\alpha_{\mathbf{m}}$ denotes the strategy of malicious bankers. The cost function takes the form

$$\begin{aligned} C_i^B = \alpha_i RT_i + (1-\alpha_i)^2 \frac{\omega A_i}{n} + (1-\alpha_i) \frac{\sum_{j \in I^p \setminus \{i\}} (1-\alpha_j)}{n} \omega \cdot A_i \\ + (1-\alpha_i) \cdot m \cdot \frac{\omega \cdot A_i}{n}. \end{aligned}$$

The last term captures the cost effect of malicious bankers. Then it must be

$$C_i^B(\alpha_i, \alpha_j; \alpha_{\mathbf{m}} = \mathbf{0}) > C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})$$

$$\begin{aligned} \alpha_i RT_i + (1-\alpha_i)^2 \frac{\omega A_i}{n} + (1-\alpha_i) \frac{\sum_{j \in I^p \setminus \{i\}} (1-\alpha_j)}{n} \omega \cdot A_i \\ + (1-\alpha_i) \cdot m \cdot \frac{\omega \cdot A_i}{n} > RT_i \end{aligned}$$

$$\begin{aligned} (1-\alpha_i) \frac{\omega \cdot A_i}{n} [m + (1-\alpha_i) + \sum_{j \in I^p \setminus \{i\}} (1-\alpha_j)] &> (1-\alpha_i) RT_i \\ \frac{\omega \cdot A_i}{RT_i} &> \frac{n}{m + (1-\alpha_i) + \sum_{j \in I^p \setminus \{i\}} (1-\alpha_j)} \\ \frac{\omega \cdot A_i}{RT_i} &> \frac{n}{m + \sum_{j \in I^p} (1-\alpha_j)}. \end{aligned}$$

□

H. PROOF OF PROPOSITION 3

We prove this viz. Proposition 2. We begin by estimating cost functions $C_i^B(\bar{\alpha}_i, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0})$ and $C_i^B(\alpha_i^*, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})$. It is easily verified that

$$C_i^B(\bar{\alpha}_i, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0}) = C_i(\bar{\alpha}_i, \alpha_j^*) = PC_i^-.$$

The bank becomes immune to systemic risk by fully complying with regulatory policy. In the same manner,

$$\begin{aligned} C_i^B(\alpha_i^*, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0}) &= \alpha_i RT_i + (1-\alpha_i)^2 \frac{\omega A_i}{n} + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n} \\ &= C_i(\alpha_i^*, \bar{\alpha}_j) + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n} \\ &= PC_i^+ + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n}. \end{aligned}$$

Following the rationale of Proposition 2, we have

$$\begin{aligned} C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0}) + C_i^B(\alpha_i^*, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0}) &\le C_i^B(\alpha_i^*, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0}) \\ &\quad + C_i^B(\bar{\alpha}_i, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0}) \\ C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0}) + C_i^B(\alpha_i^*, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0}) &\le PC_i^+ + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n} + PC_i^- \\ &\le 2APC_i + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n} \\ 1 + \frac{C_i^B(\alpha_i^*, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0})}{C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})} &\le \frac{2APC_i + (1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n}}{C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})} \\ \frac{C_i^B(\alpha_i^*, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0})}{C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})} &\le \frac{2APC_i - C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})}{C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})} \\ &\quad + \frac{(1-\alpha_i)\cdot m \cdot \frac{\omega \cdot A_i}{n}}{C_i^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})}. \end{aligned}$$

Applying summation by parts for all banks,

$$\begin{aligned} max_{\alpha^*} \frac{C^B(\alpha_i^*, \alpha_j^*; \alpha_{\mathbf{m}} = \mathbf{0})}{C^B(\bar{\alpha}_i, \bar{\alpha}_j; \alpha_{\mathbf{m}} = \mathbf{0})} &\le max_{\alpha^*} \frac{\sum_n 2APC_i - SOC}{SOC} \\ PBFA &\le \max_{\alpha^*} \left\{ \frac{2TAPC - SOC}{SOC} + \frac{\sum_{I^p} (1-\alpha_j)\cdot \frac{\omega A_i}{n}}{SOC} \right\}. \end{aligned}$$

□

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