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ON THE INFLUENCE OF NETWORK STRUCTURE ON THE RESILIENCE AND LOSSES OF FINANCIAL SYSTEMS

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ABSTRACT

We investigate resilience to defaults in financial networks, where nodes hold shares in a common set of external assets. Price fluctuations in these assets can trigger shareholder defaults, propagating through the network and causing default cascades. In particular, we conduct a series of numerical experiments to elucidate the influence of the network structure on the financial system. Our investigation explores the effects of sparsity versus connectivity, clustering, and liability heterogeneity among financial institutions. We also examine the impact of diversification and asset investment heterogeneity.

Keywords: Default cascades, financial networks, resilience.

JEL Classification Numbers: C02, C61, G01.

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1. INTRODUCTION

In recent decades, scientific interest in the vulnerability of financial networks and related systemic risk factors has grown, particularly in the aftermath of the 2007–2008 financial crisis. Understanding complex liability networks among financial institutions and their impact on the resilience of a financial system to external shocks is crucial for preventing default cascades. Despite numerous efforts, this impact remains insufficiently explored. On the one hand, a dense liability network can enhance resilience to external shocks by providing more connections to absorb a moderate shock (Allen & Gale, 2000; Freixas et al., 2000; Babus, 2016). On the other hand, dense financial networks are more fragile to intense shocks once a critical number of nodes have been affected, as more connections create additional paths for default contagion (Glasserman & Young, 2016; Acemoglu et al., 2015; Nier et al., 2007; Haldane & May, 2011; Bardoscia et al., 2017; Elliott et al., 2014).

Various models have been developed to investigate default propagation in financial networks, many inspired by the seminal work in Eisenberg & Noe (2001), which laid the foundation for extensive studies on financial contagion (Glasserman & Young, 2016; Elsinger et al., 2006; Rogers & Veraart, 2013; Acemoglu et al., 2015; Kusnetsov & Veraart, 2019; Feinstein, 2017; Hurd, 2016; Massai et al., 2022). Their model addresses the clearing of mutual liabilities when institutions default due to external shocks, providing an explicit method to compute the clearing payment matrix and total shortfall. Extensions of the Eisenberg-Noe model have incorporated crucial features of financial systems, such as prioritizing liabilities to the non-financial sector over interbank liabilities (Elsinger et al., 2006).

A critical aspect of systemic risk is asset commonality within financial networks. Alongside mutual liabilities, financial institutions hold shares in external assets. Price fluctuations in these assets can trigger complex liquidity shocks, affecting all shareholders simultaneously (Cifuentes et al., 2005; Glasserman & Young, 2016; Haldane & May, 2011; Allen et al., 2012; Banerjee & Feinstein, 2022; Amini & Feinstein, 2023), and potentially amplifying through interbank connections. Initial defaults of banks exposed to depreciating assets may trigger secondary defaults, impacting even those without direct exposure. This occurs when banks experience equity reductions due to unpaid liabilities from defaulted banks, hindering their ability to meet obligations. Recent work in Calafiore et al. (2025) quantifies the worst-case impact of such default chains, which can lead to significant losses across the financial system. Using linear programming duality, Calafiore et al. (2025) identifies the maximum price fluctuation level that ensures no bank defaults and all obligations are fulfilled. For price fluctuations exceeding this threshold, the maximum possible loss to the financial system is evaluated. Additionally, Calafiore et al. (2025) determines a critical threshold for asset price variations below which no bank becomes insolvent.

In this paper, building on the theoretical tools developed in Calafiore et al. (2025), we investigate how the structural properties of financial networks influence their resilience to shocks, focusing on the impact of network structures on worst-case system loss during price fluctuations. By structural properties, we refer to the topologies of the interbank liability graph, which depicts mutual liabilities among financial institutions, and the institution-to-asset graph, which captures how institutions diversify their investments across assets. To address this, we perform Monte Carlo-based numerical experiments to incorporate the inherent stochasticity of financial systems. These experiments allow us to analyze the effects of network structures on resilience margins and worst-case system loss, as defined in Calafiore et al. (2025), under various scenarios. Our main contributions are as follows.

First, we investigate the role of interbank connection density by comparing performance across different interbank networks, modeled as random regular digraphs with increasing degree (where each institution has liabilities to a fixed number of others). Our results indicate that very sparse interbank networks are more susceptible to default cascades; however, the benefits of increased connectivity quickly reach a saturation point. This aligns with existing literature, which highlights two opposing effects of increasing network connectivity that appear to balance each other in this scenario (Glasserman & Young, 2016; Acemoglu et al., 2015; Bardoscia et al., 2017). Second, we examine the role of the clustering coefficient by comparing performance on Watts–Strogatz networks with varying re-wiring probabilities, and thus different levels of clustering (Newman, 2018), concluding that the clustering coefficient has a negligible effect on resilience.

Third, we examine the role of heterogeneity in the interbank network by comparing results on random regular graphs with those on irregular topologies, where institutions have liabilities to varying numbers of others. Interestingly, our findings reveal a nontrivial and nonmonotone behavior: lower levels of heterogeneity appear to enhance the resilience of the financial network, while excessive heterogeneity has the opposite effect. These observations align with findings in some other model-based analyses (Caccioli et al., 2012; Battiston et al., 2012; Iori et al., 2006). Finally, we examine the network of institutional investments across different assets, analyzing how its connectivity (representing diversification) and heterogeneity impact the resilience of the financial system. As expected, our numerical results show that diversification consistently benefits financial stability. However, varying levels of diversification appear to weaken the system, indicating that a minority of institutions with poorly diversified assets can jeopardize the overall functioning of the financial network.

The rest of the paper is organized as follows. In Section 2, we present the mathematical model used for financial networks and present our research problem. Section 3 describes our main results. Section 4 concludes the paper and outlines possible future research. We summarize the key mathematical results used in resilience analysis in APPENDIX A.

2. FINANCIAL NETWORKS AND PROBLEM SETUP

We denote the set of strictly positive integers, nonnegative real numbers, and strictly positive real numbers as $\mathbb{N}_{>0}$, $\mathbb{N}_{\geq 0}$, and $\mathbb{R}_{>0}$, respectively. A vector $\boldsymbol{x}$ is represented in bold lowercase, with x_i as its ith entry. A matrix A is represented in uppercase, with a_{ij} as the jth entry of its ith row. We denote the all-0 and the all-1 vectors as $\mathbf{0}$ and $\mathbf{1}$, respectively, and the identity matrix as I, where dimensions are omitted when unnecessary. For vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ of the same dimension, $\boldsymbol{x} \geq \boldsymbol{y}$ denotes the component-wise inequalities, $x_i \geq y_i$ for all i. The relation $\leq$ is defined analogously. The operations max and min are defined element-wise; the positive part of a vector $\boldsymbol{x}$ is defined as $\boldsymbol{x}^+ = \max\{\boldsymbol{x}, \mathbf{0}\}$.

2.1. Interbank Claims Network

We consider a (finite) set $\mathcal{V}$ of $n \geq 2$ financial institutions (hereafter called *banks* for simplicity). Each bank $i \in \mathcal{V}$ has a net position with respect to the non-financial sector (referred to as its *external net position*), denoted by $c_i \in \mathbb{R}$, which represents all the bank liquidity, assets, and debts with the external sectors. The external net positions of all banks are represented by the vector $\boldsymbol{c} \in \mathbb{R}^n$. The net position can be positive or negative (Elsinger et al., 2006; Hurd, 2016), since it ultimately represents the difference between the positive external assets and debts. Note that this assumption differs from the original Eisenberg-Noe model (Eisenberg & Noe, 2001), where it is assumed that $c_i \geq 0$.

Besides external assets and liabilities, each bank may have obligations to other banks, captured by a $n \times n$ nonnegative-valued matrix $\bar{P} \in \mathbb{R}^{n\times n}_{\geq 0}$ of nominal liabilities (for instance, bank i is obligated to pay $\bar{p}_{ij}$ currency units to bank j). By definition, $\bar{p}_{ii} = 0$ (banks have no self-liability). The total liability $\bar{p}_i$ of bank i to other banks is obtained by summing the elements on the ith row of matrix $\bar{P}$, i.e.,

$$\bar{p}_i := \sum_{j\in\mathcal{V}} \bar{p}_{ij}. \tag{1}$$

Gathering these values into the vector $\bar{\boldsymbol{p}} \in \mathbb{R}^n_{\geq 0}$, one has $\bar{\boldsymbol{p}} = \bar{P}\mathbf{1}$. Similarly, the sum of the ith column of matrix $\bar{P}$, $\sum_{k\in\mathcal{V}} \bar{p}_{ki}$, represents the total liability of other banks to bank i. We introduce the stochastic matrix of *relative liabilities* $A \in [0, 1]^{n\times n}$, where each entry is defined as

$$a_{ij} = \begin{cases} \frac{\bar{p}_{ij}}{\bar{p}_i} & \text{if } \bar{p}_i > 0, \\ 1 & \text{if } \bar{p}_i = 0 \text{ and } i = j, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and represents the fraction of bank i's total liability $\bar{p}_i$ that is owed to bank j.

Such interconnections between banks naturally induce a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \bar{P})$, where a (directed) edge $(i, j) \in \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is present if and only if i has an obligation to bank j, i.e., if and only if $\bar{p}_{ij} > 0$. See, e.g., the green connections in Fig. 1. We observe that, since $\bar{P}$ is a zero-diagonal matrix, the graph $\mathcal{G}$ has no self-loops. We shall refer to the graph $\mathcal{G}$ as an *interbank network*.

2.2. Bank-to-Asset Network

Following the approach in Calafiore et al. (2025), we assume that the external net position comprises two components. First, each bank $i \in \mathcal{V}$ may hold net liquidity, denoted by c^e_i, with these values collected in the vector $\boldsymbol{c}^e := [c^e_1, \ldots, c^e_n]^\top$. Second, each bank may own shares in external assets. Regarding the latter, we assume a set of $m \in \mathbb{N}_{>0}$ liquid assets, indexed by a finite set $\mathcal{M}$. Each asset $j \in \mathcal{M}$ is characterized by a value per share $v_j \in \mathbb{R} \geq 0$, assembled into the vector $\boldsymbol{v} \in \mathbb{R}^m$. Bank i holds $s_{ij} \in \mathbb{R}$ shares of asset $j \in \mathcal{M}$,

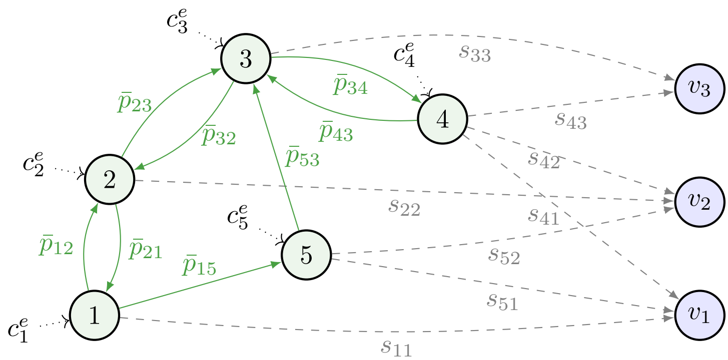

Figure 1: Illustration of a financial network with interbank network $\mathcal{G}$ (solid green edges) and external asset network $\mathcal{H}$ (dashed gray edges).

where strictly positive and negative values represent long and short positions, respectively. These entries s_{ij} are arranged into an $n \times m$ matrix $S \in \mathbb{R}^{n \times m}$.

Such an interconnection structure can also be represented as a graph, specifically, a weighted directed bipartite graph $\mathcal{H} = (\mathcal{V} \cup \mathcal{M}, \mathcal{F}, S)$, where a directed edge $(i, j) \in \mathcal{F} \subseteq \mathcal{V} \times \mathcal{M}$ exists if and only if $s_{ij} \neq 0$. We refer to this as the *external asset network* (depicted in gray in Fig. 1).

In summary, the external net position for bank $i \in \mathcal{V}$ can be written as $c_i = c_i^e + \sum_{j \in \mathcal{M}} s_{ij} v_j$, which we can write in compact form as

$$\boldsymbol{c} = \boldsymbol{c}^{\boldsymbol{e}} + S\boldsymbol{v}. \tag{3}$$

Note that the Eisenberg-Noe clearing mechanism can be extended to financial networks with cross-holdings (Elsinger, 2009), where banks can own shares in one another. Beyond the added complexity of defining a bank's equity as a function of its assets, the properties of the clearing vector (which will be introduced in Section 2.3) differ significantly from those in the original model (Eisenberg & Noe, 2001; Elsinger et al., 2006). In particular, the clearing vector is not a concave function of the vector $\boldsymbol{c}$ (Elsinger, 2009), and its computation does not reduce to a convex optimization problem like one presented in Proposition 1. Furthermore, as discussed in Elliott et al. (2014), a realistic model of banks' equities in the presence of cross-holdings should account for nonlinear failure costs, resulting in nonlinear systems of equations with multiple solutions, even when banks have no claims on each other ($\bar{p}_{ij} = 0$). Since our resilience analysis relies on characterizing clearing vectors and system loss through linear

Table 1: Model variables and parameters.

$\mathcal{V}$	set of banks
$\mathcal{M}$	set of external assets
c_i	external net position of bank $i \in \mathcal{V}$
$\bar{p}_{ij}$	liability from bank $i \in \mathcal{V}$ to bank $j \in \mathcal{V}$
$\bar{p}_i$	total debt of bank $i \in \mathcal{V}$
p_{ij}	actual clearing payment bank $i \in \mathcal{V}$ to bank $j \in \mathcal{V}$
c_i^e	net liquidity of bank $i \in \mathcal{V}$
s_{ij}	shares of asset $j \in \mathcal{M}$ owned by bank $i \in \mathcal{V}$
v_j	value per share of asset $j \in \mathcal{M}$

programming, we exclude cross-holding structures from this study, leaving them for future research.

We refer to a *financial network* as the combination of an interbank network and an external asset network, potentially including net liquidity (Fig. 1). The variables and parameters of our model are summarized in Table 1.

2.3. Clearing Vectors and System Loss

In regular operations, each bank $i \in \mathcal{V}$ has a nominal external net position $c_i = \bar{c}_i$ sufficiently large so that the bank is able to pay its debts at the end of the period, possibly by liquidating some of its external assets, in other words, the nominal equity of the bank, defined as the sum of the nominal external net position and the credits to other banks minus its liabilities, is nonnegative:

$$\bar{w}_i := \bar{c}_i + \sum\nolimits_{k \in \mathcal{V}} \bar{p}_{ki} - \bar{p}_i \geq 0, \quad \forall i \in \mathcal{V}. \tag{4}$$

However, if a financial shock impacts the network, the actual external net position of some bank i may decrease from its nominal value to $c_i < \bar{c}_i$, e.g., due to a decrease in an external asset's value, as we shall see below. In such a scenario, bank i may be unable to meet its obligations even if its debtors fulfill their liabilities completely, as its actual equity defined as

$$w_i := c_i + \sum_{k \in \mathcal{V}} \bar{p}_{ki} - \bar{p}_i \tag{5}$$

appears to be negative. We say that bank i incurs a *primary default* because its inability to fulfill liabilities is caused directly by the external shock. In this case, bank i should fulfill its liabilities to the best of its capacity, prioritizing obligations to the external sector, followed by those to other banks. As a result, the *actual payment* p_{ij} from i to j, in general, may be less than its nominal value $\bar{p}_{ij}$, decreasing the asset side of bank j's balance sheet. This, in turn, may lead to *secondary defaults*: A bank j that could meet its obligation $\bar{p}_j$ if its debtors fulfilled their obligations in full will actually default if those obligations are not met and

$$c_j + \sum_{k \in \mathcal{V}} p_{kj} - \bar{p}_j < 0. \tag{6}$$

In the latter case, the actual claims of some other banks on j must be reduced from $\bar{p}_{jk}$ to $p_{jk} < \bar{p}_{jk}$, to tertiary defaults and further cascades. In this way, even a single shock can trigger multiple bank defaults, causing a substantial shortfall across the financial system.

Eisenberg & Noe (2001), further generalized in Elsinger et al. (2006), proposed a constructive method for determining the actual clearing payments $p_{ij} \leq \bar{p}_{ij}$ between banks in the event of one or more defaults. One assumption imposed in Eisenberg & Noe (2001) is the equal priority of debt claims, implying proportional sharing in case of default (the pro-rata division rule). Thus, if $p_i = \sum_{j \in \mathcal{V}} p_{ij}$ is the total clearing payment from bank i to other banks, then $p_{ij} = a_{ij} p_i$ for all j. In view of this rule, it suffices to determine the vector of clearing payments $\boldsymbol{p} \in \mathbb{R}^n$, composed of the values p_i. Two additional rules relate to the banks' actual equities after fulfilling the clearing payments and can be expressed as follows Elsinger et al. (2006):

$$p_i = \begin{cases} \bar{p}_i & \text{if } \; c_i + \sum_{k \neq i} a_{ki} p_k \geq \bar{p}_i, \\ c_i + \sum_{k \neq i} a_{ki} p_k & \text{if } \; \bar{p}_i > c_i + \sum_{k \neq i} a_{ki} p_k \geq 0, \\ 0 & \text{if } \; c_i + \sum_{k \neq i} a_{ki} p_k < 0. \end{cases} \tag{7}$$

These equations imply, on one hand, the *limited liability rule*: a bank's total payment must not exceed its net residual value, so that its equity remains nonnegative (except in cases of insolvency, where equity is negative even without interbank payments). On the other hand, they impose the *absolute debt priority rule*: bank owners receive no value unless all nominal liabilities are fully cleared. Rewriting these equations in compact form leads to the definition of a clearing vector (Elsinger et al., 2006).

Definition 1. *A clearing vector for the interbank network* $(\mathcal{V}, \mathcal{E}, \bar{P})$ *with vector* *of net external positions* $\boldsymbol{c}$ *is a solution* $\boldsymbol{p}$ *to the following equation*

$$\boldsymbol{p} = \min(\bar{\boldsymbol{p}}, (\boldsymbol{c} + A^\top \boldsymbol{p})^+). \tag{8}$$

It is known (this follows, e.g., from the Knaster-Tarski fixed point theorem (Eisenberg & Noe, 2001; Elsinger et al., 2006; Glasserman & Young, 2016)) that at least one clearing vector always exists. Furthermore, the set of clearing vectors contains both a least element $\boldsymbol{p}_*(\boldsymbol{c})$ and a greatest element $\boldsymbol{p}^*(\boldsymbol{c})$, that is, for every solution to Eq. (8) one has

$$\boldsymbol{p}_*(\boldsymbol{c}) \leq \boldsymbol{p} \leq \boldsymbol{p}^*(\boldsymbol{c}). \tag{9}$$

In the generic situation, the least and the greatest clearing vectors coincide $\boldsymbol{p}_*(\boldsymbol{c}) = \boldsymbol{p}^*(\boldsymbol{c})$, and the clearing vector is uniquely determined by $\boldsymbol{c}$. Necessary and sufficient criteria for uniqueness are provided in Csóka & Herings (2024); Calafiore et al. (2024) for the case where $\boldsymbol{c} \geq 0$ and in Hurd (2016); Massai et al. (2022) for the general case $\boldsymbol{c} \in \mathbb{R}^n$.

In exceptional cases where the uniqueness cannot be guaranteed, we will assume that the banks use the greatest clearing vector $\boldsymbol{p}^*(\boldsymbol{c})$, which provides the minimal value of *system loss* (Glasserman & Young, 2016) (the total shortfall across the financial network) $L(\boldsymbol{p})$ among all clearing vectors. Here,

$$L(\boldsymbol{p}) = \mathbf{1}^\top(\bar{\boldsymbol{p}} - \boldsymbol{p}) = \|\bar{\boldsymbol{p}} - \boldsymbol{p}\|_1 = \sum_i (\bar{p}_i - p_i). \tag{10}$$

For a given vector $\boldsymbol{c}$, the greatest clearing vector (and the system loss associated to it) can be determined, e.g., through iterations of a monotone operator (Calafiore et al., 2025) or a modified fictitious default algorithm (Rogers & Veraart, 2013). However, since we aim to evaluate the *worst-case* system loss over a class of unknown vectors $\boldsymbol{c}$, these characterizations appear to lack of relevance. Instead, we use an extremal characterization of the greatest clearing vector (Proposition 1), which, as discussed in APPENDIX A, applies to "moderate" shocks that do not lead to insolvencies. Clearly, if $\boldsymbol{c} \geq \bar{\boldsymbol{c}}$, then no defaults are present due to Eq. (4), and thus $\boldsymbol{p}^*(\boldsymbol{c}) = \bar{\boldsymbol{p}}$, and there is no loss in the financial system $L(\boldsymbol{p}^*(\boldsymbol{c})) = 0$.

2.4. Resilience margin and worst-case loss

In this paper, we focus on financial shocks arising from fluctuations in actual asset prices. Specifically, we assume that the actual price vector $\boldsymbol{v}$ in Eq. (3)

can fluctuate and is expressed as

$$\boldsymbol{v} = \bar{\boldsymbol{v}} + \boldsymbol{\delta}, \tag{11}$$

where $\boldsymbol{\delta} \in \mathbb{R}^m$ represents the vector of price fluctuations, and $\bar{\boldsymbol{v}}$ denotes the nominal value. For instance, in contexts where fluctuations might be captured by stochastic processes, the nominal value $\bar{\boldsymbol{v}}$ can be interpreted as the expected value of the price for share. The vector of actual external net positions is given by

$$\boldsymbol{c} = \bar{\boldsymbol{c}} + S\boldsymbol{\delta}, \tag{12}$$

where $\bar{\boldsymbol{c}} := \boldsymbol{c}^{\boldsymbol{e}} + S\bar{\boldsymbol{v}}$ is the vector of nominal external net positions, and $S\boldsymbol{\delta}$ captures the impact of price fluctuations. In the absence of fluctuations $\boldsymbol{\delta} = \mathbf{0}$, the condition from Eq. (4) is assumed to hold, which is reformulated as follows.

Assumption 1. *The quantities A, $\bar{\boldsymbol{p}}$, S, $\boldsymbol{c}^{\boldsymbol{e}}$, $\bar{\boldsymbol{v}}$ are such that $\boldsymbol{c}^{\boldsymbol{e}} + S\bar{\boldsymbol{v}} + A^\top \bar{\boldsymbol{p}} - \bar{\boldsymbol{p}} \geq 0$.*

In presence of price fluctuations, as has been discussed in the previous section, a natural measure of the loss in the financial system is given by

$$\eta(\boldsymbol{\delta}) := L(\boldsymbol{p}^*(\bar{\boldsymbol{c}} + S\boldsymbol{\delta})), \tag{13}$$

which represents the loss corresponding to the greatest clearing vector compatible with the vector of the banks' actual external net positions $\boldsymbol{c} = \bar{\boldsymbol{c}} + S\boldsymbol{\delta}$. While this loss can be computed for each fluctuation vector $\boldsymbol{\delta}$, the price fluctuations in practice are unpredictable. This uncertainty leads to the problem of evaluating the *worst-case loss*, under the assumption that the magnitude of the fluctuation vector $\boldsymbol{\delta}$ is bounded in some norm: $\|\boldsymbol{\delta}\| \leq \varepsilon$. Different norms can be used to study different aspects of the price fluctuation. For instance, the ℓ_∞ norm ($\|\boldsymbol{\delta}\|_\infty = \max_{i\in\mathcal{M}} |\delta_i|$) captures independent variation of each asset price, while a bound in the ℓ_1 norm ($\|\boldsymbol{\delta}\|_1 = \sum_{i\in\mathcal{M}} |\delta_i|$) poses a constraint on the sum of the amplitudes of all perturbations. In other words, the worst-case loss is defined as the maximum value in the optimization problem

$$\begin{aligned} \eta_{wc}(\varepsilon) = \max\nolimits_{\delta} \quad & \eta(\boldsymbol{\delta}) \\ \text{s.t.:} \quad & \|\boldsymbol{\delta}\| \leq \varepsilon, \end{aligned} \tag{14}$$

where $\eta(\boldsymbol{\delta})$ is defined in Eq. (13). It can be shown (Calafiore et al., 2025) that $\eta(\boldsymbol{\delta})$ is a *convex* function, making its *maximization* over a convex set a nontrivial problem. However, this problem can be addressed for the ℓ_∞ and ℓ_1

norms, as described above, assuming that ε is not too large. As demonstrated in Calafiore et al. (2025), under these conditions, the worst-case loss computation reduces to a max-min problem that can be solved using LP duality. The method from Calafiore et al. (2025) enables the determination of the following financial network's characteristics.

First, one can determine the *resilience* margin (Calafiore et al., 2025), i.e., the maximum fluctuation magnitude ε^* such that no fluctuation vector $\boldsymbol{\delta}$ with $\|\boldsymbol{\delta}\| \leq \varepsilon^*$ leads to defaults. In other words, for $\boldsymbol{c} = \bar{\boldsymbol{c}} + S\boldsymbol{\delta}$, banks can fully meet their debt obligations while keeping all equities from Eq. (5) nonnegative.

Second, the method from Calafiore et al. (2025) determines the maximum level of admissible fluctuation ε_{ub} such that price fluctuations with $\|\boldsymbol{\delta}\| \leq \varepsilon_{ub}$ cause a shock that does not result in insolvencies. More formally, for the vector of external net positions $\boldsymbol{c} = \bar{\boldsymbol{c}} + S\boldsymbol{\delta}$, a clearing vector exists under which all liabilities to the external sector can be fully cleared, even if defaults occur with respect to interbank liabilities. The optimal, in the sense of the system loss, clearing vector is the greatest clearing vector $\boldsymbol{p}^*(\boldsymbol{c})$, which, in this case, can be found from an LP (see Proposition 1). The number ε_{ub}, called the *insolvency margin*, can also be found from a similar linear program (Calafiore et al., 2025, Theorem 6).

Finally, for each $\varepsilon \in [\varepsilon^*, \varepsilon_{ub}]$, it is possible to compute the worst-case system loss $\eta_{wc}(\varepsilon)$ for fluctuations satisfying $\|\boldsymbol{\delta}\| \leq \varepsilon$, under the ℓ_1 and ℓ_∞ norms. In these two important cases, the optimization problem in Eq. (14) reduces to a single linear program (for the ℓ_1 norm) or a set of independent linear programs (for the ℓ_∞ norm), as detailed in Propositions 2 and 3). These results form the basis for our numerical study.

2.5. Research problem

In the remainder of this paper, we build on the computational results summarized above to conduct an extensive numerical campaign exploring how the structure of the interbank network and the external asset network influence the resilience of the financial network as a whole. Specifically, in the first set of studies, we focus on the interbank network $\mathcal{G}$ and examine key features such as connectivity, clustering, and degree distribution. These results expand some of the findings preliminarily analyzed in Zino et al. (2025), where a simpler scenario is studied in the absence of an explicit external asset network. In the second set of studies, our analysis shifts to the external asset network $\mathcal{H}$. In order to guarantee that Assumption 1 is satisfied, for each institution $i \in \mathcal{V}$, we assume that

$$\bar{c}_i = \bar{p}_i - \sum_{k \in \mathcal{V}} \bar{p}_{ki} + \xi_i, \tag{15}$$

where ξ_i is a realization of a random variable uniformly distributed in $[0, 100]$, with each realization independent of the others. Hence, in the absence of fluctuations no defaults are possible, and all institutions have the same expected nominal equity $\bar{w}_i$ from Eq. (4), with different realizations due to stochasticity.

We use the tools from Propositions 2 and 3 to compute the worst-case loss (η_{wc}) for increasing values of the maximal norm of the admissible fluctuation not exceeding the insolvency margin, $\varepsilon \leq \varepsilon_{ub}$. To achieve this, we generate the worst-case loss curve by solving linear programs at different (evenly spaced) levels of fluctuation within the range $[0, \varepsilon_{ub}]$.

In each study, we generate financial networks (i.e., $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \bar{P})$ and $\mathcal{H} = (\mathcal{V} \cup \mathcal{M}, \mathcal{F}, S)$) with the desired characteristics. In order to obtain statistically significant results we introduce stochasticity in the network formation process and we average the simulation outcomes over each random scenario. Specifically, in each of our studies we generate 100 independent instances of each network, and all the results presented hereafter are obtained by averaging the outputs of these 100 independent instances. All simulations are performed using MATLAB, and the code is available at https://github.com/lzino90/financial.

3. RESULTS

This section provides the details of our numerical experiments, together with our numerical results.

3.1. Impact of interbank network

In this section, we present and discuss the results of our numerical investigation of the role of the interbank network on the resilience of a financial system and the worst-case loss in the presence of defaults. To perform such a study, we consider a network of $n = 200$ banks. The interbank network is generated depending on the specific feature under analysis, as detailed below. The external asset network is instead kept fixed throughout all the numerical experiments in this section, and it comprises $m = 4$ external assets. Each bank $i \in \mathcal{V}$ has shares in either 1, 2, 3, or 4 assets, with uniform probability across all possible configurations. Then, the number of shares in each asset is sampled uniformly at random and the resulting values are normalized, so that $s_{i1} + s_{i2} + s_{i3} + s_{i4} = 100$ for all $i \in \mathcal{V}$.

3.1.1. Network connectivity

We start by investigating the impact of the network connectivity, viz. of the density of the interbank network. As discussed in the introduction, the impact of such a feature is complex, as it apparently leads to two contrasting effects, whereby the resilience could be either increased (Allen & Gale, 2000; Freixas et al., 2000; Babus, 2016), or decreased (Glasserman & Young, 2016; Acemoglu et al., 2015; Nier et al., 2007; Haldane & May, 2011; Bardoscia et al., 2017).

To investigate such a problem, we proceed as follows. We assume that the network $\mathcal{G}$ is a (weighted) directed regular random graph with out-degree equal to d (i.e., each bank has liabilities to exactly d other banks). We explore the role of the network connectivity by computing the worst-case loss as a function of the norm of the fluctuation ε (computed using our theoretical tools), for different values of $d \in \{1, \dots, 10\}$. Clearly, larger values of d model networks with a denser connectivity pattern. Specifically, for each value of d, we generate a directed regular random graph of out-degree d. Such a network is generated as follows. For each node $i \in \mathcal{V}$, we sample a d-uple of nodes uniformly at random in $\mathcal{V} \setminus \{i\}$, with each institution independent of the others. Then, i is connected to these nodes. The total liability is split evenly across the (out)-neighbors, with each entry equal to $100/d$, so that $\bar{p}_i = 100$ for all $i \in \mathcal{V}$. Note that, in general, we obtain a directed network, due to the independent mechanisms through which each node selects its (out-)neighbors.

The results of our numerical experiments are represented in Fig. 2. First, we observe that the results obtained with the two different norms are qualitatively consistent. Predictably, fluctuations bounded in the ℓ_1 norm (i.e., with bounds on the total sum of the fluctuations and not just on its maximal value) lead to smaller total worst-case losses than the one bounded in the ℓ_∞ norm. Interestingly, we observe that the network connectivity impacts the performance of the financial network. In fact, curves corresponding to very sparse networks ($d = 1$ or $d = 2$) are shorter (i.e., the insolvency margin ε_{ub} is smaller) and they are above the others. This means that such network structures have low resilience and typically lead to larger worst-case losses with respect to more

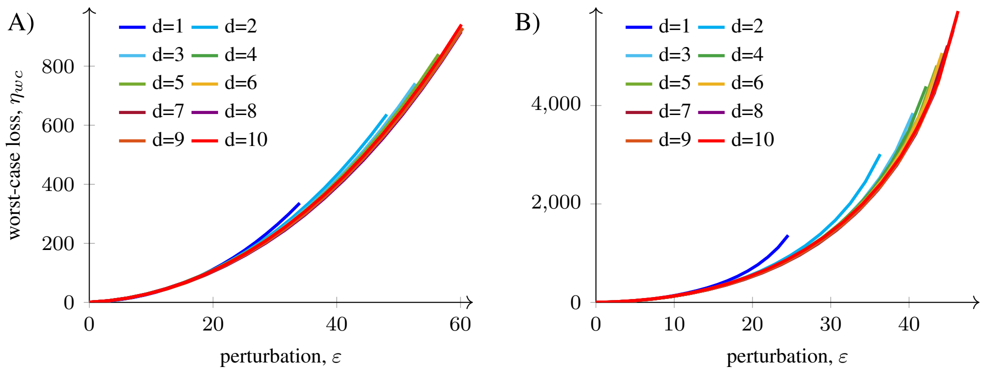

Figure 2: Impact of the network connectivity, computed numerically over 100 independent realizations of random regular graphs with different out-degree d, with fluctuations bounded in the A) ℓ_1 and B)ℓ_∞ norm.

connected structures.

However, it is worth noticing that the marginal benefit of increasing the network connectivity quickly decreases as the connectivity increases. In fact, for a network with $n = 200$ banks, one can still see some very minimal improvements up to $d = 5$; but then all curves when $d > 6$ are practically indistinguishable. Hence, we conclude that, while very poorly connected networks are weak and prone to default cascades, the beneficial effect of increasing the network connectivity seems to quickly decline.

3.1.2. Clustering

Many real-world financial systems are characterized by a phenomenon, whereby it is more likely that pair of entities that share common connections are, in turn, connected one to the other (Onnela et al., 2004; Rakib et al., 2021). In network science, such phenomenon is called clustering. Here, we investigate how clustering may impact the financial system. To this aim, we generate each instance of the interbank network as a Watts–Strogatz small-world network with average degree $d = 4$ (within these networks undirected, in- and out- degrees coincide), with a different re-wiring probability,[1] which allows us to generate

[1] Watts–Strogatz small-world networks are obtained starting from a regular lattice (which is highly clustered), and re-wiring edges in a stochastic fashion in order to add long-range interactions, which decrease the network clustering and the network diameter (Newman, 2018).

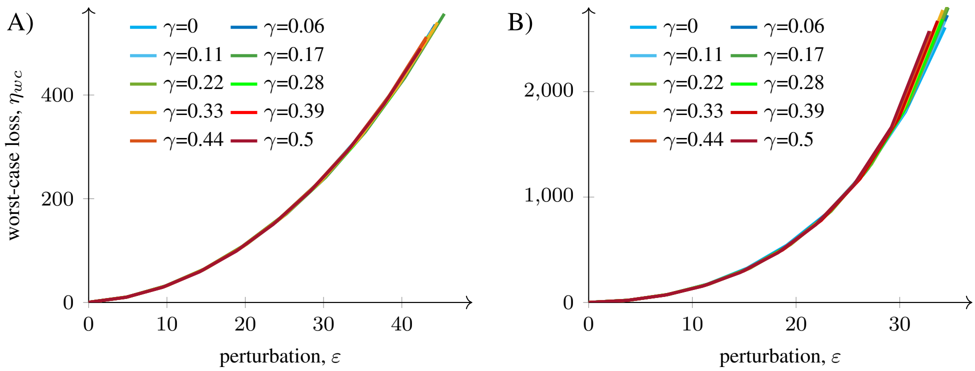

Figure 3: Impact of clustering, computed numerically over 100 independent realization of random regular graphs with different clustering coefficient γ, with fluctuations bounded in the A) ℓ_1 and B) ℓ_∞ norm.

networks with the same average degree, but with different levels of clustering. In fact, there is a one-to-one relation between the re-wiring probability and the network clustering coefficient, which is denoted by γ and represents the number of triadic closures in the network (in plain words, the larger γ, the higher the level of clustering). See, Newman (2018) for more details. Then, similar to the previous study, for each desired level of clustering we generate realizations of such a network structure and we set nonzero liabilities to $\bar{p}_{ij} = 25$. This makes the simulation comparable to the earlier scenarios where $\bar{p}_i = 100$.

Our numerical results, represented in Fig. 3, suggest that the level of clustering in the financial network has a very marginal impact on its performance. In particular, we observe that the worst-case loss seems not to be affected by the level of clustering. However, we observe that highly clustered networks tend to have slightly smaller values of the insolvency margin ε_{ub}. This means that highly clustered networks are slightly less resilient to large price fluctuations. This marginal impact may be due to the presence of two contrasting phenomena. On the one hand, once one or multiple defaults occur within a cluster of banks, all the other banks in that cluster have large exposure to them, possibly leading to further defaults. On the other hand, the other banks in the network would have little exposure to such banks, and are thus less likely to have defaults.

3.1.3. Heterogeneity

Another important feature of real-world financial networks is the presence of heterogeneity across different banks. To investigate the impact of heterogeneity, we compare the performances of the system on network structures with different levels of heterogeneity. Specifically, we consider random graphs with the following out-degree distributions: i) homogeneous, with each node having the same number of neighbors $d = 4$; ii) moderately heterogeneous, with each node having number of neighbors d_i sampled from a zero-truncated Poisson distribution (Ross, 2019) with mean 4; and iii) highly heterogeneous, with each node having number of neighbors d_i sampled from a Zipf distribution with mean equal to 4. All networks are then generated by sampling, for each node i, the set of (out-)neighbors as a d_i-uple of nodes uniformly at random, each node independent of the others. Similar to the other studies, we set the nonzero liabilities to $\bar{p}_{ij} = 100/d = 25$, to make this scenario comparable with the previous ones.

Our results, reported in Fig. 4, depict a nontrivial behavior. In fact, on the one hand, heterogeneity seems always detrimental to the overall network resilience reducing the value of the insolvency margin ε_{ub}, especially for fluctuations bounded in the ℓ_∞ norm. This means that it becomes easier for default cascades to lead to insolvency to the external sector when the system is heterogeneous. On the other hand, the impact on the worst-case loss is nonmonotonic. In fact, consistently across the two different norms, it seems that smaller levels of heterogeneity (green curve) might slightly decrease the worst-case loss, consistent with the results in other model-based studies (Caccioli et al., 2012). However, such a beneficial effect could be jeopardized by further increasing the heterogeneity (see, e.g., the red curve), consistent with the results in Glasserman & Young (2015).

We conjecture that a reason for such a nontrivial behavior can be that nodes with lower out-degree, which may be less robust in the face of price fluctuations, typically generate smaller cascades when they fail, yielding a smaller total loss. However, in highly heterogeneous networks, the default of some marginal nodes can ultimately lead one of the core banks to a default, yielding a larger cascade and, ultimately, a larger total loss. These results, which are consistent with results from other studies on the nontrivial and nonlinear impact of diversity and heterogeneity in the interbank network (Iori et al., 2006; Battiston et al., 2012), suggest that finding an optimal level of diversity in the network structure

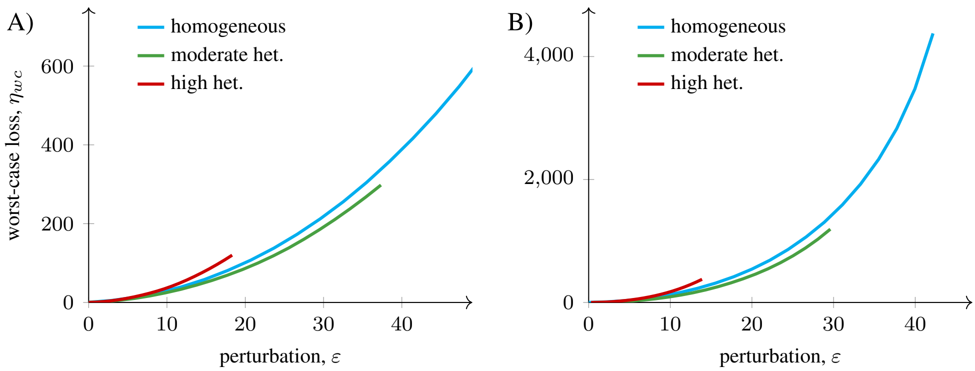

Figure 4: Impact of out-degree heterogeneity, computed numerically over 100 independent realizations of random regular graphs (homogeneous, cyan curve), Erdos-Renyi random networks (low heterogeneity, green curve) and Albert–Barabasi scale-free networks (high heterogeneity, red curve), with fluctuations bounded in the A) ℓ_1 and B) ℓ_∞ norm.

is indeed a key but nontrivial problem, which should be further investigated.

3.2. Impact of external asset network

Finally, we analyze how the characteristics of the external asset network impact the resilience of the financial network and its worst-case loss in case of price fluctuations. To perform such studies, we fix the characteristics of the interbank network across all numerical experiments to be a directed regular random graph with $n = 200$ banks (see Section 3.1.1 for the details on how such networks are generated), each one connected with other $d = 4$ banks with total liability $\bar{p}_i = 100$ for each $i \in \mathcal{V}$ split evenly across the 4 (out-)neighbors. Then, we generate external asset networks with different features, to assess their impact on the system. In all these networks, we consider $m = 8$ external assets. In this analysis, we consider only worst-case losses when considering fluctuations bounded in the ℓ_1 norm, while fluctuations bounded in the ℓ_∞ norm are omitted, being less interesting. In fact, ℓ_∞ norm captures the maximal fluctuation among all prices. This decouples the effect of the different assets, reducing the effect of the network.

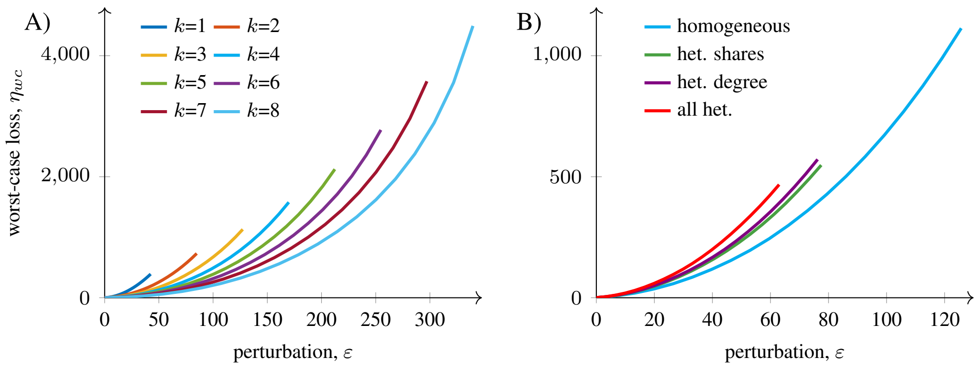

Figure 5: Impact of the external asset network, computed numerically over 100 independent realization of the financial network, with fluctuations bounded in the ℓ_1 norm. In A), we consider an external asset network with different connectivity (i.e., different degree k); in B), we compare the homogeneous scenario (cyan) with respect to heterogeneous scenarios in the number of shares across assets (green), the number of assets (violet), and both features (red).

3.2.1. Diversification

In our first study, we investigate the impact of diversification in the assets. To this aim, we consider scenarios in which each bank has shares in exactly k external assets (sampled as a k-uple for each bank, independent of the others), with a total amount of 100 shares divided evenly across the banks. In our analysis we vary the value of k in the discrete interval from 1 to $m = 8$ and we use the theoretical tools developed in Proposition 2 to compute the worst-case loss when considering fluctuations bounded in the ℓ_1 norm. Our results are reported in Fig. 5A).

Consistent with intuition and with the economic theory, we conclude that diversification is beneficial in financial networks. In fact, in scenarios in which each bank divides its shares among more external assets, the financial network becomes more resilient to larger fluctuations in price and, in correspondence of the same level of fluctuation, lower worst-case losses are obtained.

3.2.2. Heterogeneity

Finally, we investigate the role of diversity and heterogeneity in the external asset network. Specifically, we investigate such an effect along two directions, viz. by allowing banks to have i) a different number of shares in their assets and ii) shares in a different number of assets. The results of our analysis are reported in Fig. 5B).

Specifically, we compare a scenario in which each bank has shares in exactly $k = 3$ assets with 33.3 shares in each asset (cyan curve); a second scenario in which the total 100 shares are divided uniformly at random across the $k = 3$ assets (green curve); a third scenario in which each bank has 100 shares divided evenly across a random number of assets[2] (violet curve); and, finally, a fourth scenario in which both features of heterogeneity are included (red curve).

Our results suggest that, while diversification across the available assets is beneficial to the financial network, having heterogeneity in how banks allocate their shares (both in the number of assets and in the distribution of shares per asset) seems always disadvantageous. We conjecture that this is due to the inherent weakness of banks with poor diversification (either because they invest in few assets, or they allocate most of their shares in a single asset). These banks are thus more subject to a default, and may act as the weakest link in a chain, leading to default cascades in the entire financial network.

4. CONCLUSION

In this paper, we used an extended version of the Eisenberg–Noe model (Eisenberg & Noe, 2001; Elsinger et al., 2006) that accounts for external assets and, building on the theoretical developments in Calafiore et al. (2025), we investigated a critical problem in financial systems: *how does the structure of the financial network impact its resilience to default cascades?* To perform such a study, we systematically put forward a campaign of numerical experiments in which we assessed the resilience of the financial system under different assumptions regarding its network structure, and we computed the total worst-case loss in the presence of defaults due to fluctuations of the assets' prices.

The results of our numerical experiments suggest that both the network that describes the pattern of mutual liabilities between banks and the one describing how banks have shares in the different assets impact the performances of the financial network. In particular, we have determined that some features of the interbank network have a key role, such as connectivity and heterogeneity. In fact, very sparse networks are more likely to be subject to default cascades,

[2] Precisely, the number of assets is equal to a zero-truncated binomial random variable (Ross, 2019) with mean equal to 3.

whereas a moderate level of heterogeneity seems to be beneficial for decreasing worst-case losses in the presence of defaults. These observations corroborate similar results obtained with different methods and modeling approaches (Caccioli et al., 2012; Battiston et al., 2012; Iori et al., 2006). On the contrary, other features, such as the presence of clusters, seem to play only a marginal role in the emergent behavior of the financial network. Concerning the interactions of banks with assets, predictably, we have found that diversification is key to reducing the financial risk. On the contrary, heterogeneity in how banks invest in the external assets seems detrimental to the financial system, since it may generate some weak links in the network.

Besides elucidating the role of the network structure on the resilience of financial systems and on the worst-case loss in the presence of defaults, the results of our numerical experiments pave the way for several lines of future research. First, our results on the impact of heterogeneity in the interbank network depict a nontrivial scenario, whereby moderate levels of heterogeneity might be beneficial in reducing potential losses in financial networks. Further numerical studies should be pursued in order to determine the optimal level of heterogeneity and understand whether other features of the degree distribution may further impact the system's performances. Second, our numerical experiments rely on the theoretical tools developed in Calafiore et al. (2025). A future extension of the theoretical framework from Calafiore et al. (2025) to account for further features of financial systems, e.g., cross holdings between banks, the presence of illiquid assets, and defaults and bankruptcy costs (Cifuentes et al., 2005; Banerjee & Feinstein, 2022), would produce novel theoretical tools, which should be used to check the robustness of our findings and extend our numerical experiments. Moreover, the extension of the linear program framework above the insolvency margin, i.e., when fluctuations may lead to the insolvency of one or more banks, should be pursued by deriving novel theoretical results to compute the worst-case loss when a clearing vector does not exist. Third, the network features investigated in this paper are based on the characteristics empirically observed in financial systems (Onnela et al., 2004; Rakib et al., 2021). A relevant extension of this work is the analysis of case studies of real-world financial systems and the validation of our findings against the results obtained with other stress test methodologies (Amini et al., 2012; Battiston & Martinez-Jaramillo, 2018).

APPENDIX A. MATHEMATICAL METHODS

In this appendix, we summarize the mathematical framework established in Calafiore et al. (2025) to compute the maximum admissible fluctuation in the price of the asset before the financial network witnesses some defaults, and the worst-case loss, when fluctuations exceed the threshold.

A.1. Extremal Property of the Greatest Clearing Vector

The key step in characterizing the worst-case loss is outlined in the following proposition, which demonstrates that the minimal loss and the greatest clearing vector corresponding to the net external position vector $\boldsymbol{c}$ can often be determined through a simple linear program.

Proposition 1. *(Calafiore et al., 2025, Proposition 2) Consider the LP*

$$\begin{array}{rl} \min_{\boldsymbol{p}\in\mathbb{R}^n} & \mathbf{1}^\top(\bar{\boldsymbol{p}}-\boldsymbol{p}) \\ s.t.: & \mathbf{0} \leq \boldsymbol{p} \leq \bar{\boldsymbol{p}} \\ & \boldsymbol{c} + A^\top \boldsymbol{p} \geq \boldsymbol{p}. \end{array} \tag{16}$$

If the problem in Eq. (16) is feasible, then its optimal solution $\boldsymbol{p}^*$ is unique and coincides with the greatest clearing vector $\boldsymbol{p}^*(\boldsymbol{c})$. Moreover, under $\boldsymbol{p}^*$, all liabilities held by the external sector are paid. On the contrary, if the problem in Eq. (16) is infeasible, then there will exist banks which are insolvent with respect to the external sector for any admissible clearing vector $\boldsymbol{p}$.

When we consider the special case where $\boldsymbol{c} \geq 0$ (making the LP automatically feasible), we observe that this result appears in numerous works (Eisenberg & Noe, 2001; Glasserman & Young, 2016; Calafiore et al., 2024); the general case can be proved similarly.

The constraints in Eq. (16) ensure that the vector of interbank payments $\boldsymbol{p} \in \mathbb{R}^n$, under the proportionality rule $p_{ij} = p_i a_{ij}$, adheres to the limited liability principle: after clearing liabilities to the external sector and fulfilling these payments, the banks' equities remain nonnegative. As demonstrated in Calafiore et al. (2025), the feasibility of Eq. (16) is guaranteed for shocks whose magnitude does not exceed a critical threshold $\|\boldsymbol{\delta}\| \leq \varepsilon_{ub}$, which can be explicitly determined. The greatest clearing vector $\boldsymbol{p}^*(\boldsymbol{c})$ in the Eisenberg-Noe model is optimal among all such payment vectors, minimizing the system loss.

The infeasibility of Eq. (16) indicates that the net positions of the banks, $\boldsymbol{c}$, do not allow for clearing external liabilities without rendering at least one bank insolvent. This infeasibility can be addressed in several ways. One approach is to introduce additional binary variables to indicate whether a bank can remain solvent, as demonstrated in a similar extremal characterization of clearing vectors in Ararat & Meimanjan (2023, Corollary 4). Insolvent banks pay nothing to other banks and only partially fulfill their external liabilities. However, extending our results to MILP optimization presents a significant challenge. Moreover, defining system loss in the case of insolvencies constitutes a standalone research topic. Alongside the shortfall of banks, the loss to the external sector must be considered, as it can exacerbate the situation for banks in subsequent periods. Additionally, the costs associated with insolvencies must also be accounted for. On the other hand, by redefining the concept of system loss, one can consider other modifications to the LP in Proposition 1 to address potential infeasibility. For instance, the key constraint could be relaxed and replaced with a penalty term in the cost function

$$\begin{array}{rl} \min_{\boldsymbol{p}\in\mathbb{R}^n} & \mathbf{1}^\top(\bar{\boldsymbol{p}}-\boldsymbol{p})+\boldsymbol{\mu}^\top(\boldsymbol{p}-\boldsymbol{c}-A^\top\boldsymbol{p})^+ \\ \text{s.t.:} & \mathbf{0}\leq\boldsymbol{p}\leq\bar{\boldsymbol{p}}. \end{array} \tag{17}$$

Here, $\boldsymbol{\mu}$ is a positive vector of penalty weights. Essentially, the “residual” $s_i = p_i - c_i - (A^\top \boldsymbol{p})_i$, when positive, can be interpreted as additional liquidity provided to a problematic bank by a regulating authority, similar to the model in (Calafiore et al., 2022). The penalty weights can significantly influence the optimum, and their selection requires separate analysis.

A.2. The Worst-Case Loss Computation

By applying the strong duality theorem in linear programming (Calafiore et al., 2025, Theorem 5), an explicit characterization of the worst-case loss can be derived in the case of ℓ_1 and ℓ_∞ norms of the asset price fluctuation vector.

Proposition 2. *(Calafiore et al., 2025, Section 4.1) For any level of fluctuation smaller than the insolvency margin $\varepsilon \leq \varepsilon_{ub}$, the worst-case loss across all asset price variation vectors $\boldsymbol{\delta}$ with $\|\boldsymbol{\delta}\|_1 \leq \varepsilon$ is*

$$\begin{array}{rl} \eta_{wc} = \max_{i=1,\dots,m}\max_{\boldsymbol{\beta},\boldsymbol{\lambda}\geq 0} & (\mathbf{1}-\boldsymbol{\beta})^\top\bar{\boldsymbol{p}}-\bar{\boldsymbol{c}}^\top\boldsymbol{\lambda}+\varepsilon \boldsymbol{z_i}^\top\boldsymbol{\lambda} \\ s.t.: & \boldsymbol{\beta}-\mathbf{1}+(I-A)\boldsymbol{\lambda}\geq 0, \end{array} \tag{18}$$

where $\boldsymbol{z_i}^\top$ is the component-wise absolute value of the i-th row of matrix S.

In other words, the worst-case loss can be efficiently computed by simply solving m independent linear programs, one associated with each asset.

Corollary 1. *The following properties concerning the set of arguments* $(i^*, \boldsymbol{\lambda}^*, \boldsymbol{\beta}^*)$ *that maximize the objective function in Eq.* (18) *hold true:*

1. *The argument* i^* *determines the asset which would have the largest impact on the financial network due to fluctuations in its value. More precisely, the vector* $\boldsymbol{\delta}$ *is defined as*

$$\delta_i = \begin{cases} 0 & \text{if } i \neq i^*, \\ -\varepsilon & \text{if } i = i^*. \end{cases} \tag{19}$$

2. *The argument* $\boldsymbol{\lambda}^*$ *identifies the set of banks that have a primary default in the worst-case scenario:* $\lambda_j^* > 0$ *if and only if* j *has a primary default.*

Proof. Item 1) has been proved in Calafiore et al. (2025). To prove item 2), from the constraint in Eq. (18) for the generic jth bank, we get $\beta_j + \lambda_j - 1 + (A\boldsymbol{\lambda})_j \geq 0$, where the last term depends on some entries of $\boldsymbol{\lambda}$, but not on λ_j as $a_{jj} = 0$. Hence, the constraint requires the sum $\beta_j + \lambda_j$ to be greater than some term independent of these two variables. Let us now focus on the contribution associated with β_j and λ_j to the objective function, which is equal to

$$(1 - \beta_j)\bar{p}_j + \lambda_j(\varepsilon s_{i^*j} - \bar{c}_j) = \bar{p}_j + \lambda_j(\varepsilon s_{i^*j} - \bar{c}_j) + \beta_j(-\bar{p}_j). \tag{20}$$

with the constraints that $\beta_j + \lambda_j \geq 1 + \sum_{k\neq j} a_{ij}\lambda_k$. Clearly, increasing β_j has always a negative impact. Moreover, keeping constant the sum of β_j and λ_j (which should be larger than $1 + \sum_{k\neq j} a_{jk}\lambda_k$), it is more convenient to have $\lambda_j > 0$ if and only if $\varepsilon s_{i^*j} - \bar{c}_j > \bar{p}_j$, i.e., if bank j has a primary default due to the perturbation $\boldsymbol{\delta}$ defined in Eq. (19). Hence,

$$\lambda_j^* = \begin{cases} 1 + \sum_{k\neq j} a_{jk}\lambda_k^* & \text{if } j \text{ has a primary default,} \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

which implies

$$\beta_j^* = \begin{cases} 0 & \text{if } j \text{ has a primary default,} \\ 1 + \sum_{k\neq j} a_{jk}\lambda_k^* & \text{otherwise,} \end{cases} \tag{22}$$

yielding the second claim. □

The case of ℓ_∞ norm leads to an even simpler formulation, where a single linear program is obtained, as detailed in the following.

Proposition 3. *(Calafiore et al., 2025, Section 4.2) For any level of fluctuation smaller than the insolvency margin $\varepsilon \le \varepsilon_{ub}$, the worst-case loss across all price fluctuation vectors with $\|\boldsymbol{\delta}\|_\infty \le \varepsilon$ is equal to*

$$\begin{array}{rl} \eta_{wc} = \max_{\beta,\lambda\ge 0} & (\mathbf{1}-\boldsymbol{\beta})^\top \bar{\boldsymbol{p}} - \bar{\boldsymbol{c}}^\top \boldsymbol{\lambda} + \varepsilon \mathbf{1}^\top \tilde{S}^\top \boldsymbol{\lambda} \\ s.t.: & \boldsymbol{\beta} - \mathbf{1} + (I - A)\boldsymbol{\lambda} \ge 0, \end{array} \tag{23}$$

where $\tilde{S}$ is the component-wise absolute value of matrix S.

Remark 1. *Also for Proposition 3, the argument that maximizes the objective function in Eq. (23) provides information on the banks that are affected by such a fluctuation, as in Corollary 1.*

Remark 2. *For both Propositions 2 and 3, when the level of fluctuation is smaller than the resilience margin, i.e., $\varepsilon < \varepsilon^*$, the solution of the linear programs in Eq. (18) and Eq. (23) gives $\eta_{wc} = 0$, since no defaults occur.*

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